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A sufficient condition for cyclability in directed graphs

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Abstract

Given a strongly connected digraph D of order n and a subset S of $V(D)$, we prove that if any two nonadjacent vertices of S have degree sum at least 2n-1, there is a directed cycle in D that contains all the vertices of S. This result generalizes the Theorem of Meyniel on hamiltonicity.

Résumé

Soit D un graphe orienté fortement connexe d'ordre n et S un sous ensemble de $V(D)$. On démontre que si pour toute paire de sommets non adjacents de S la somme des degrés vaut au moins 2n-1, alors il existe dans D un cycle orienté qui contient tous les sommets de S. Ce résultat généralise le Théorème de Meyniel sur les cycles hamiltoniens.

Keywords: directed graphs, cycle, cyclability, hamiltonian graphs.
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1 Introduction

For convenience, terminology and notations will be given in details in Section 2. We will just recall the notion of cyclability for undirected graphs.

A set S of vertices in an undirected graph G is said to be *cyclable in* G if G contains a cycle through all the vertices of S. This definition was first introduced by K. Ota in [9]. Clearly, if putting $S = V(G)$, we get as corollaries classical results on hamiltonicity. There are many well known conditions which guarantee the cyclability of a set of vertices in a graph. Most of them can be seen as restrictions of hamiltonian conditions to the considered set of vertices. Let us cite for example the following ones that involve minimum degree or minimum degree sum for pairs of nonadjacent vertices.

Theorem 1.1 *[10, (Shi)] Let G be a 2-connected graph of order n and $S \subset V(G)$. If $d(x) \geq \frac{n}{2}$ for all vertices $x \in S$, then S is cyclable in G.*

Theorem 1.2 *[10, (Shi)] Let G be a 2-connected graph of order n and $S \subset V(G)$. If $d(x) + d(y) \geq n$ for any two nonadjacent vertices $x \in S$, $y \in S$, then S is cyclable in G.*

Notice that the 2-connectivity assumption has be weakened in later articles. Also Theorems 1.1 and 1.2 generalize the classical Theorems on hamiltonicity of Dirac and Ore, respectively (in which theorems the 2-connected assumption is not mentioned since it is in fact implied by the degree condition).

We can also mention the more general result on k-LTW-sequences ([4]), where a non-negative real sequence $a = (a_1, a_2, ..., a_{k+1})$ is called a *k-LTW-sequence* if $a_1 \leq 1$ and for any $i_1, i_2, ..., i_l \in \{2, 3, ..., k+1\}$, $\sum_{j=1}^{l} i_j \leq k+1 \Longrightarrow \sum_{j=1}^{l}(a_{i_j} - 1) \leq 1$. For this result, we need to recall the following notations. Given a graph G and an independent set X of p vertices in $V(G)$,

$$S_i(X) = \{u \in V : |N(u) \cap X| = i\}, \text{ for } i = 0, 1, ..., p,$$

and

$$N^i(X) = \{u \in V(G) : \min_{v \in X} d_G(u, v) = i\},$$

where $d_G(u, v)$ is the distance between u and v in G. Clearly $N(X) = N^1(X)$, $N^0(X) = X$ and $S_0(X) = X \cup (\cup_{t \geq 2} N^t(X))$. Let $n(X) : = |N^0(X) \cup N^1(X) \cup N^2(X)| = n - |\cup_{t>2} N^t(X)|$.

Theorem 1.3 *[4, (Favaron, Flandrin, Li, Tian,)] Let G be a k-connected graph ($k \geq 2$) of order n and $S \subseteq V(G)$. If there exist some t, $1 \leq t \leq k$ and some t-LTW-sequence $a = \{a_1, a_2, ..., a_{t+1}\}$ such that for each independent set $X \subseteq S$ with $t+1$ vertices, we have*

$$\sum_{i=1}^{t+1} a_i|S_i(X)| > n(X) - 1,$$

then S is cyclable in G.

For directed graphs there are not in litterature as many conditions as for undirected graphs that guarantee hamiltonicity and sufficient degree conditions for hamiltonicity also assume the digraphs to be strongly connected (or strong). The more classical ones are the three following.

Theorem 1.4 *[5, (Ghouila-Houri)] Let D be a strong digraph of order n. If $d(x) \geq n$ for all vertices $x \in V(D)$, then D is Hamiltonian.*

Theorem 1.5 *[11, (Woodall)] Let D be a strong digraph of order n. If $d^+(x) + d^-(y) \geq n$ for all pairs of vertices x and y such that there is no arc from x to y, then D is Hamiltonian.*

Theorem 1.6 *[8, (Meyniel)] Let D be a strong digraph of order n. If $d(x) + d(y) \geq 2n - 1$ for all pairs of non-adjacent vertices in D, then D is Hamiltonian.*

Theorems 1.4 and 1.5 are in fact corollaries of Theorem 1.6. In this paper we prove that Theorem 1.6 has a cyclable version and even better, where a set of vertices of a digraph D is said to be *cyclable in D* if D contains a directed cycle trough all the vertices of S.

Theorem 1.7 *Let D be digraph of order n and $S \subset V(D)$. If D is S-strong and if $d(x) + d(y) \geq 2n - 1$ for all pairs of non-adjacent vertices in S, then S is cyclable in D.*

This result admits clearly the following theorem as a corollary, and consequently, making $S = V(D)$, Meyniel's Theorem and therefore Woodall's and Ghouila-Houri's Theorems are also corollaries of Theorem 1.7.

Theorem 1.8 *Let D be a strong digraph of order n and $S \subset V(D)$. If $d(x) + d(y) \geq 2n - 1$ for all pairs of non-adjacent vertices in S, then S is cyclable in D.*

In Section 3 we will prove two technical lemmas, Theorem 1.7 will be proved in Section 4 and we will give some conjectures and concluding remarks in Section 5.

2 Terminology and notations

For standard terminology we refer to [3] and for complementary results on directed graphs to [1]. When considering directed graphs (or digraphs), cycles and paths are implicitely directed. Given a vertex x of directed path P or a directed cycle C, we use the notations x^+ and x^- for the successor and the predecessor of x (on P or C) according to the orientation and in case of ambiguity, we precise P or C as a subscript (that is x_P^+,...). The length of a path is equal to its number of arcs and a digraph is *hamiltonian* if it contains a cycle through all its vertices. We assume that graphs and digraphs have no loops nor multiple edges or arcs respectively.

Let D be a digraph and x, y be distinct vertices in D. If there is an arc from x to y in D, we say that x *dominates* y and use the notation $x \rightarrow y$ to denote this. For a vertex x in $V(D)$ and a subgraph H in D, the *in-degree*, $d_H^-(x)$, of x with respect to H is the number of vertices in H dominating x. The *out-degree*, $d_H^+(x)$, is the number of vertices in H dominated by x. We also write $N_H^-(x)$ ($N_H^+(x)$) to denote the set of the vertices of H dominating x (dominated by x), respectively. The degree of x with respect to H is $d_H(x) = d_H^-(x) + d_H^+(x)$. When $H = D$, the subscript H will be omitted.

Given two distinct vertices a and b in $V(D)$, a directed path P from a to b is called an (a, b)*-path.* If x and y are vertices of P, the subpath of P from x to y is denoted by $P[x, y]$. Let C be is a directed cycle in D containing vertices x and y; analogously $C[x, y]$ denotes the subpath of C from x to y and we define an (x, y)−path P to be a *C-bypass* if $|V(P)| \geq 3$ and $V(P) \cap V(C) = \{x, y\}$. We call the length of the path $C[x, y]$ the *gap of P with respect to C*. Given a directed path $P = v_1v_2...v_k$ in D and a vertex v in $V(D) - V(P)$, we say that v *is insertible in P* if there exists i, $1 \leq i \leq k - 1$ such that $v_i \rightarrow v$ and $v \rightarrow v_{i+1}$. If v is insertible in P, then D clearly contains the path $P' = v_1...v_ivv_{i+1}...v_k$.

The digraph D is said *strongly connected* (or just *strong*) if there exists an $(x,y)-$path and a $(y,x)-$path in D for any pair of distinct vertices x,y in D. If we only consider the set $S \subset V(D)$, we denote the vertices of S by S-vertices and the number of S-vertices in a path or a cycle is called its S-length. We also define a notion of strong connectivity restricted to S as follows : D is S-strongly connected (or S-strong) if for any pair x,y of distinct S-vertices there exists an $(x,y)-$path and a $(y,x)-$path in D.

3 Technical Lemmas

We first give a lemma that is in fact a refinement of the classical "bypass lemma" (see for example [6]) when taking in account a subset S of $V(D)$.

Lemma 3.1 *Let D be a digraph of order n and $S \subset V(D)$, $S \neq \emptyset$. Assume that D is S-strong and satisfies for any pair of nonadjacent vertices x,y in S the degree condition $d(x)+d(y) \geq 2n-1$. If C is a cycle in D of maximum S-length and s a S-vertex of $V(D)-V(C)$, then D contains a C-bypass through s.*

Proof: Let $C = c_1c_2...c_pc_1$ and $S \cap C = \{s_1, s_2, ..., s_t\}$ in that order on C. It is easy to show that C contains at least two S-vertices since any two S-vertices are on a cycle of length at most 4 by our degree condition (cf for example [6]). Since s is not in C, let us consider seperately the case when s has a neighbor on C or not.

Case 1. $N_C(s) \neq \emptyset$

We can assume without lost of generality that there is a vertex u in C such that $s \rightarrow u$. Let x be any vertex of $S \cap V(C)$. As D is S-strong, there exists a path from x to s, and consequently an $u's$-path P with $u' \in C$ and $V(P) \cap V(C) = \{u'\}$. If u' is different from u, then we have a C-bypass through s, so assume $u' = u$ and P is chosen with minimum length from u to s. Put $P' = P[u^+, s]$ and $R = V(D) - (V(C) \cup V(P'))$. The sets $V(C)$, $V(P')$ and R make a partition of $V(D)$ and $|V(C)| + |V(P')| + |R| = n$. Let s' be a vertex of $S \cap V(C)$ different from u. We suppose that s and s' are not adjacent, otherwise we can obtain a C-bypass through s with extremities u and s'. We now consider $d(s) + d(s')$ and assume there is no C-bypass through s.

- Clearly $N_C^+(s) = \{u\}$ and so $d_C^+(s) = 1$. Also $d_C^-(s) = 1$ or 0 according to the fact that u dominates s or not.

- Vertex s' can be adjacent to any vertex in C but itself, therefore $d_C^-(s') + d_C^+(s') \leq 2(|V(C)| - 1)$.

- From our minimality assumption on P, s is dominated by exactly one vertex of P' if $|V(P')| \geq 2$ and otherwise we are in the case when u dominates s. In any case, we have $d_C^-(s) + d_{P'}^-(s) = 1$.

- Vertex s can dominate every vertex but itself in P' which implies $d_{P'}^+(s) \leq |V(P')| - 1$.

- Similarly, every vertex in P' but s can dominate s', and $d_{P'}^-(s') \leq |V(P')| - 1$.

- Vertex s' dominates no vertex in P' otherwise we would get a C-bypass and $d_{P'}^+(s') = 0$.

- For the same reason, for every vertex r in R, we cannot have $s \rightarrow r$ and $r \rightarrow s'$ nor $s' \rightarrow r$ and $r \rightarrow s$, which implies $d_R^+(s) + d_R^-(s') \leq |R|$ and $d_R^-(s) + d_R^+(s') \leq |R|$.

We now compute $d(s) + d(s') = d_C(s) + d_C(s') + d_{P'}(s) + d_{P'}(s') + d_R(s) + d_R(s')$ and from the above observations obtain the majoration

$$d(s)+d(s') \leq 1+2(|V(C)|-1)+1+|V(P')|-1+|V(P')|-1+0+|R|+|R| = 2n-2,$$

a contradiction.

Case 2. $N_C(s) = \emptyset$

Similarly to Case 1, from the S-strong connectivity of D and the fact that C contains (at least two) vertices of S, there are necessarily an su_1-path P_1 and an u_2s-path P_2 where u_1 and u_2 are two vertices of C such that $P_1 \cap V(C) = \{u_1\}$ and $P_2 \cap V(C) = \{u_2\}$. Let us choose those two pathes P_1 and P_2 such that $|V(P_1) \cup V(P_2)|$ is minimum. To simplify further computations, put $V(P_1[s^+, u_1^-]) \cap V(P_2[u_2^+, s^-]) = L$, $V(P_1[s^+, u_1^-]) - L = \Pi_1$, $V(P_2[u_2^+, s^-]) - L = \Pi_2$ and $R = V(D) - (V(C) \cup \Pi_1 \cup \Pi_2 \cup L \cup \{s\})$. Notice that R, $V(C)$, Π_1, Π_2, L and $\{s\}$ form a partition of $V(D)$, and that $n = |R| + |V(C)| + |\Pi_1| + |\Pi_2| + |L| + 1$.

In the case when $V(P_1)$ and $V(P_2)$ are disjoint, we obtain a C-bypass through s, so we only consider the case when they are not. Let us choose a vertex s' in $S \cap V(C)$ which is by hypothesis nonadjacent to s.

As in Case 1, we will compute $d(s)+d(s')$, assuming there is no C-bypass through s.

- Clearly $d_C^+(s)+d_C^-(s)=0$.

- Vertex s' can be adjacent to any vertex in C but itself, therefore $d_C^-(s')+d_C^+(s')\leq 2(|V(C)|-1)$.

- From our minimality assumption on $P_1\cup P_2$, s dominates exactly one vertex on $P_1[s^+,u_1^-]$ and can be dominated only by vertices of $P_1[s^+,u_1^-]$ that are not in P_2 and also possibly by the vertex of L which is the closest to s. Analogous observations hold for s and P_2. More precisely :

$d_{\Pi_1}^-(s)\leq|\Pi_1|$ and $d_{\Pi_2}^+(s)\leq|\Pi_2|$,
$d_{\Pi_1}^+(s)=1$ if $s_{P_1}^+\notin L$ and 0 otherwise,
$d_{\Pi_2}^-(s)=1$ if $s_{P_2}^-\notin L$ and 0 otherwise,
$d_L^+(s)=0$ if $s_{P_1}^+\notin L$ and 1 otherwise,
$d_L^-(s)=0$ if $s_{P_2}^-\notin L$ and 1 otherwise.

Consequently $d_{\Pi_1\cup\Pi_2\cup L}(s)\leq|\Pi_1|+|\Pi_2|+2$ in any case.

- Similarly, we can prove $d_{\Pi_1\cup\Pi_2\cup L}(s')\leq|\Pi_1|+|\Pi_2|+2$ (distinguishing several cases according to the fact that the closest vertex to C on $P_1[s^+,u_1^-]$ and $P_2[u_2^+,s^-]$ is in L or not).

- For the same reason as in case 1, $d_R^+(s)+d_R^-(s')\leq|R|$ and $d_R^-(s)+d_R^+(s')\leq|R|$.

Summing all the above inequalities, we obtain

$$d(s)+d(s')\leq 0+2(|V(C)|-1)+2(|\Pi_1|+|\Pi_2|+2)+2|R|=$$

$$(2n-2)+(2-2|L|)\leq 2n-2 \text{ if } |\mathrm{L}|\geq 1.$$

In the case when $|L|=0$, necessarily $u_1=u_2=u$, otherwise $P_2[u_2,s]P_1[s,u_1]$ is a C-bypass through S. Now we choose $s'\in S\cap V(C)$ different from this vertex u. We get as above $d_{\Pi_1\cup\Pi_2}(s)\leq|\Pi_1|+|\Pi_2|+2$ but $d_{\Pi_1\cup\Pi_2}(s')\leq|\Pi_1|+|\Pi_2|$ since $d_{\Pi_1}^-(s')=d_{\Pi_2}^+(s')=0$ from the absence of a C-bypass through s. We also obtain $d(s)+d(s')\leq 2n-2$ and in any case we have a contradiction with our degree assumption which achieves the proof of Lemma 3.1. □

We now prove the following result on insertibility.

Lemma 3.2 *Let D be a digraph and $P = v_1 v_2 ... v_k$ a path in D with k vertices. Let v be a vertex of $V(D) - V(P)$ which is not insertible in P. Then $d_P(v) \leq k - 1 + d^+_{v_1}(v) + d^-_{v_k}(v)$.*

Proof: The proof is quite easy. Because of the noninsertibility of v in P, for any i, $1 \leq i \leq k-1$, we have $d^-_{v_i}(v) + d^+_{v_{i+1}}(v) \leq 1$. Since $d_P(v) = d^+_{v_1}(v) + d^-_{v_k}(v) + \Sigma_{i=1}^{i=k-1}(d^-_{v_i}(v) + d^+_{v_{i+1}}(v))$, we then have the proclaimed result. □

4 Proof of Theorem 1.7

The outline of the proof follows the usual way for hamiltonian problems in digraphs.

Assume that D fulfills the assumptions of Theorem 1.7 but S is not cyclable in D. Let us choose a cycle $C = x_1 x_2 ... x_t$ that contains as many vertices of S as possible and a S-vertex s in $V(D) - V(C)$. From Lemma 3.1, we know that D contains a C-bypass through s, $P = u_1 ... s ... u_m$. Without loss of generality, let $u_1 = x_1$, $u_m = x_{\alpha+1}$, $\alpha < t$. We assume that the pair (C, P) is *gap-minimal* through s, that is for every other pair Z, R, where Z is a cycle so that $V(Z) = V(C)$ and R a Z-bypass through s, the gap of R with respect to Z is not smaller than the gap of P. We also assume that, under the condition of gap minimality, P is chosen as short as possible through s and put $P' = P[u_2, u_{m-1}]$ and $R = V(D) - (V(C) \cup V(P'))$. Clearly $|V(C)| + |V(P')| + |V(R)| = n$. Let $C_1 = C[x_2, x_\alpha]$, $C_2 = C[x_{\alpha+1}, x_1]$. Necessarily from the non cyclability of S, C_1 and C_2 both contain at least one vertex of S. Let s' be the first S-vertex of C_1, that is the closest to x_1 following C, which cannot be adjacent to s.

We are going to compute the degree sum of the vertices s and s'.

- Since (C, P) is gap minimal, s has no neighbors in C_1 and $d_{C_1}(s) = 0$.
- Vertex s' is not adjacent to itself and so $d_{C_1}(s') \leq 2(|V(C_1)| - 1)$.
- By Lemma 3.2 and since s is not insertible in C_2 by the definition of C, we know that $d_{C_2}(s) \leq |V(C_2)| - 1 + d^+_{x_{\alpha+1}}(s) + d^-_{x_1}(s)$.
- By the minimality assumption on P, s dominates no vertex in $P[s^+, u_{m-1}]$ but possibly its successor if $s^+ \neq x_{\alpha+1}$. Analogously, no vertex in $P[u_2, s^-]$ dominates s but possibly s^- if $s^- \neq x_1$. Hence, in any case, since s is not adjacent to itself, $d_{P'}(s) \leq |P'| + 1 - d^+_{x_{\alpha+1}}(s) - d^-_{x_1}(s)$.

- Because of gap minimality, s' dominates no vertex in $P'[u_2, s]$ and is not dominated by any vertex in $P'[s, u_{m-1}]$, and so $d_{P'}(s') \leq |P'| - 1$.

- For the same reason, for every vertex r in R, we cannot have $s \rightarrow r$ and $r \rightarrow s'$ nor $s' \rightarrow r$ and $r \rightarrow s$, and so $d_R(s) + d_R(s') \leq 2|R|$.

Using now the above inequalities, we obtain

$$2n - 1 \leq d(s) + d(s') \leq 0 + 2(|V(C_1)| - 1) + |V(C_2)| - 1$$

$$+d^+_{x_{\alpha+1}}(s) + d^-_{x_1}(s) + d_{C_2}(s') + |P'| + 1 - d^+_{x_{\alpha+1}}(s) - d^-_{x_1}(s) + |P'| - 1 + 2|R|.$$

That is $d_{C_2}(s') \geq 2|V(C_2)| + 2$.

By Lemma 3.2, s' is insertible in C_2. All the S-vertices of C_1 can be successively inserted in C_2 in a similar way. Considering the new path obtained from C_2 and all the S-vertices of C_1, together with the C-bypass through s, we then obtain a cycle that contains all the s-vertices that were in C and the vertex s, a contradictions that achieves the proof of Theorem 1.7. □

5 Concluding remarks

In [7], Y. Manoussakis proves the following :

Theorem 5.1 *[7, (Manoussakis)] Suppose D is strong and satisfies the following condition for every tripple $x, y, z \in V(D)$ such that x and y are non-adjacent: If there is no arc from x to z, then $d(x) + d(y) + d^+(x) + d^-(z) \geq 3n-2$. If there is no arc from z to x then $d(x)+d(y)+d^-(x)+d^+(z) \geq 3n-2$. Then D is Hamiltonian.*

We put as a question to know if this result has a cyclable version.

We also are interested into the results of Bang-Jensen, Gutin and Li ([2]) concerning special pairs of vertices that do not satisfy the out-LSD or the in-LSD property. Let us recall that the vertices x and y satisfy *the out-LSD property* (*the in-LSD property*) if either x and y are adjacent or there is no $z \in V(D) - \{x, y\}$ which dominates both x and y (is dominated by both x and y). In other words, if some pair of vertices x and y does not satisfy the out-LSD (in-LSD) property, then x and y are not adjacent and there exists a vertex $z \in V(D) - \{x, y\}$ which dominates both x and y (is dominated by both x and y).

Bang-Jensen, Gutin and Li have proved the two Theorems and made the two conjectures that are following.

Theorem 5.2 *Let D be a strong digraph. Suppose that, for every pair of vertices x, y which does not satisfy the out-LSD property, either $d(x) \geq n$ and $d(y) \geq n-1$ or $d(x) \geq n-1$ and $d(y) \geq n$. Then D is Hamiltonian.*

Theorem 5.3 *Let D be a strong digraph. Suppose that, for every pair of vertices x, y which does not satisfy either the out-LSD property or the in-LSD property, $d^+(x)+d^-(y) \geq n$ and $d^-(x)+d^+(y) \geq n$. Then D is Hamiltonian.*

Conjecture 5.4 *Let D be a strong digraph. Suppose that, for every pair of vertices x, y which does not satisfy either the out-LSD property or the in-LSD property, $d(x) + d(y) \geq 2n - 1$. Then D is Hamiltonian.*

Conjecture 5.5 *Let D be a strong digraph. Suppose that, for every pair of vertices x, y which does not satisfy the out-LSD property, $d(x)+d(y) \geq 2n-1$. Then D is Hamiltonian.*

Those results only involve special pairs of nonadjacent vertices in the graph. If considering a special subset of vertices, we put the question of the adaptation of the notions of out-LSD and in-LSD and if together with the strong S-connectivity, cyclability results can be obtained.

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