

*L3 Mention Informatique  
Parcours Informatique et MIAGE*

# Génie Logiciel Avancé

## UML/MOAL II

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# Plan of the Chapter

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- Semantics of MOAL Constraints
  - Class Invariants
  - Pre- and Post-Conditions
- Other applications of MOAL:
  - ... in sequence diagrams
  - ... in state machines

# Recall:

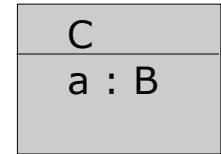
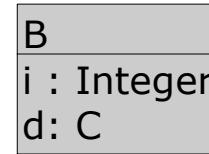
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- ❑ MOAL is logics used to make UML diagrams more precise
- ❑ it comprises
  - typed sets, lists, and some base types
  - classes and objects from UML class diagrams
  - subtyping and casts
  - a semantics for path navigation and associations.

# Recall: Object Attributes

- Objects represent structured, typed memory in a state  $\sigma$ . They have **attributes**.

They can have class types.

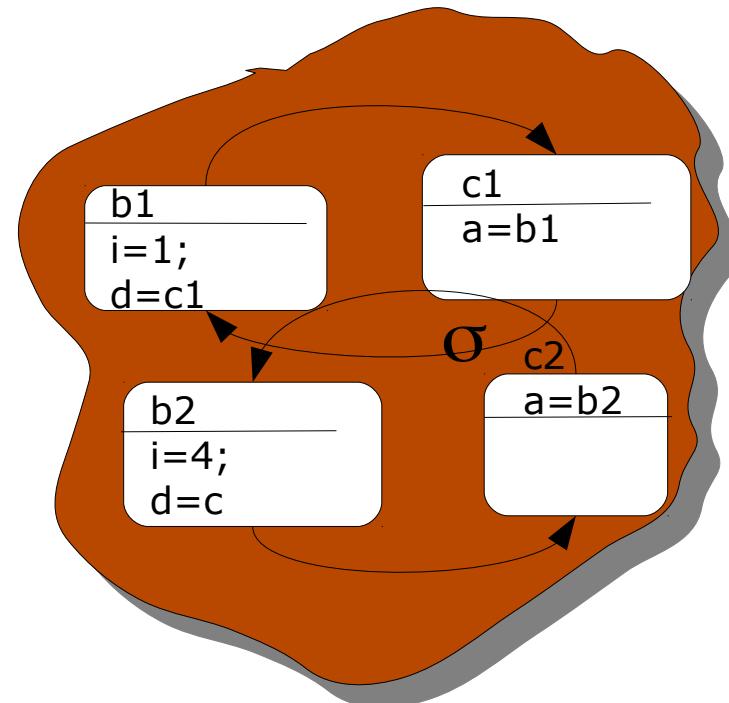
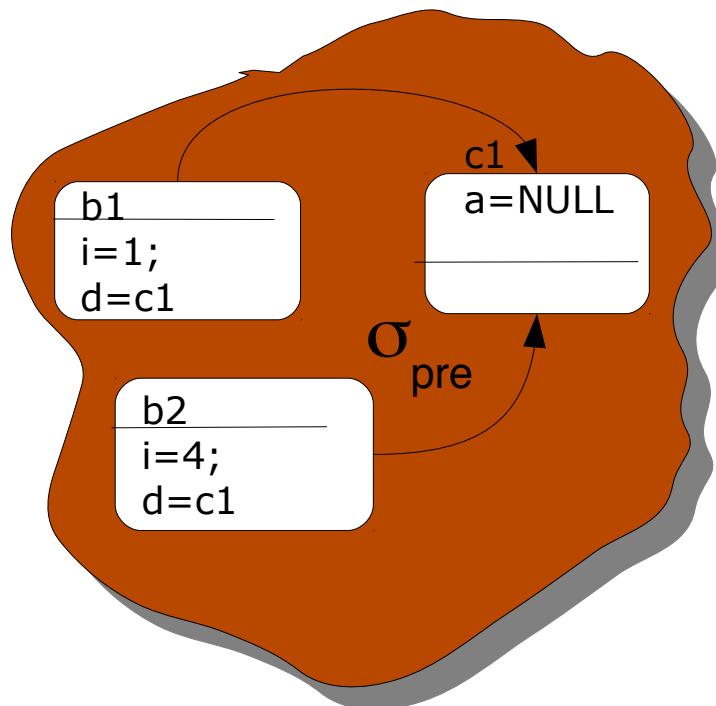


- Reminder: In class diagrams, this situation is represented traditionally by Associations (equivalent)



# Syntax and Semantics of Object Attributes

- Example:  
attributes of class type in states  $\sigma'$  and  $\sigma$ .



# Recall Object Attributes

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- Object assessor functions are „dereferentiations of pointers in a state“
- Accessor functions of class type are **strict** wrt. NULL.
  - $\text{NULL}.\text{d} = \text{NULL}$
  - $\text{NULL}.\text{a} = \text{NULL}$
  - Recall that navigation expressions depend on their underlying state:  
 $b1.\text{d}(\sigma_{\text{pre}}).\text{a}(\sigma_{\text{pre}}).\text{d}(\sigma_{\text{pre}}).\text{a}(\sigma_{\text{pre}}) = \text{NULL}$   
 $b1.\text{d}(\sigma).\text{a}(\sigma).\text{d}(\sigma).\text{a}(\sigma) = b1 \quad !!!$

(cf. Object Diagram pp 28)

# Recall Object Attributes

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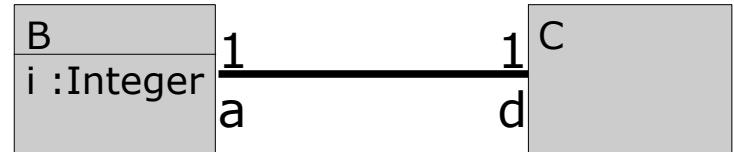
- Object assessor functions are „dereferentiations of pointers in a state“
- Accessor functions of class type are **strict** wrt. NULL.
  - $\text{NULL.d} = \text{NULL}$
  - $\text{NULL.a} = \text{NULL}$
  - The  $\sigma$  convention allows to write :

$\text{old(b1.d.a.d.a)} = \text{NULL}$   
 $\text{b1.d.a.d.a} = \text{b1} \quad !!!$

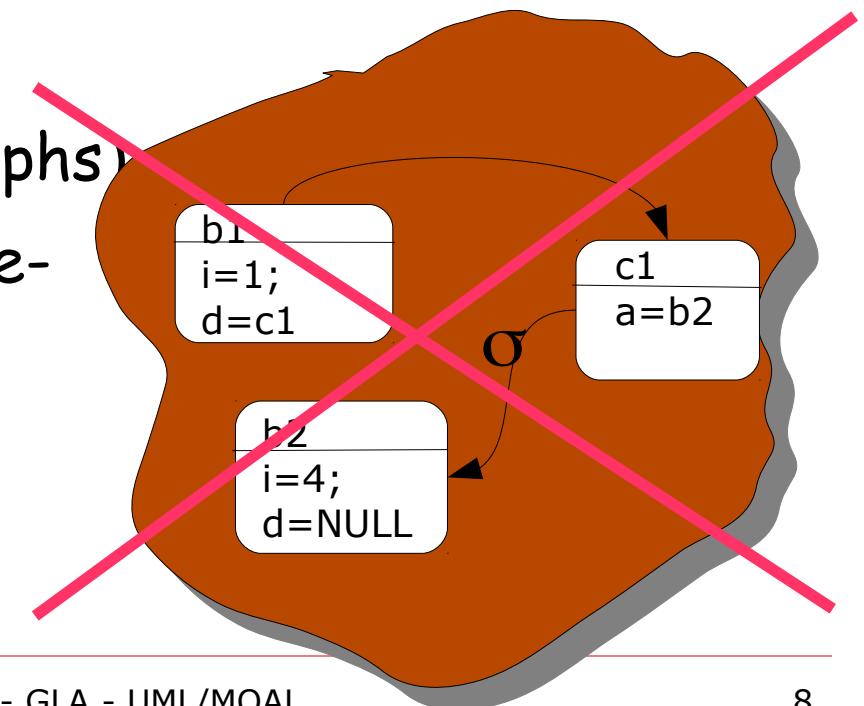
(cf. Object Diagram pp 28)

# Recall Object Attributes

- ❑ Note that associations are meant to be « relations » in the mathematical sense.



Thus, states (object-graphs) of this form do not represent an association:



# Recall Object Attributes

- This is reflected by 2 « association integrity constraints ».  
For the 1-1-case, they are:



- definition  $\text{ass}_{B.d.a} \equiv \forall x \in B. x.d.a = x$
- definition  $\text{ass}_{C.a.d} \equiv \forall x \in C. x.a.d = x$

# Recall Object Attributes

- ❑ Attributes can be List or Sets of class types:

|             |
|-------------|
| B           |
| i : Integer |
| d: Set(C)   |

|             |
|-------------|
| C           |
| a : List(B) |

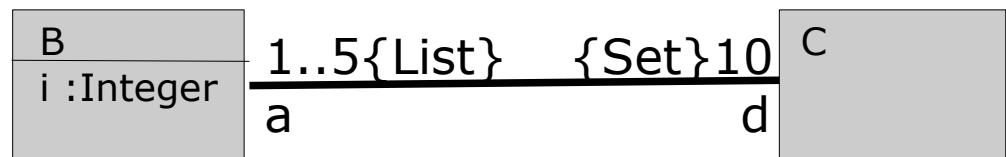
- ❑ Reminder: In class diagrams, this situation is represented traditionally by Associations (equivalent)



- ❑ In analysis-level Class Diagrams, the type information is still omitted; due to overloading of  $\forall x \in X. P(x)$  etc. this will not hamper us to specify ...

# Recall Object Attributes

- Cardinalities in Associations can be translated canonically into MOCL invariants:



- definition  $\text{card}_{B.d} \equiv \forall x \in B. |x.d| = 10$
- definition  $\text{card}_{C.a} \equiv \forall x \in C. 1 \leq |x.a| \leq 5$

# Strictness of Collection Attributes

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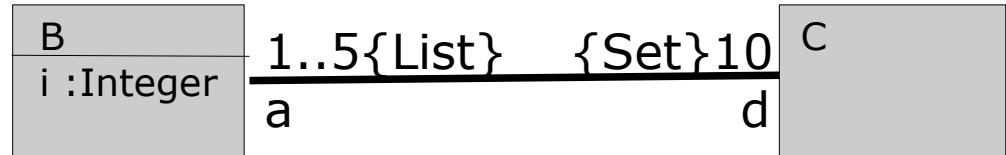
- ❑ Accessor functions are defined as follows for the case of NULL:



- `NULL.d = {}` -- mapping to the neutral element
- `NULL.a = []` -- mapping to the neural element.

# Syntax and Semantics of Object Attributes

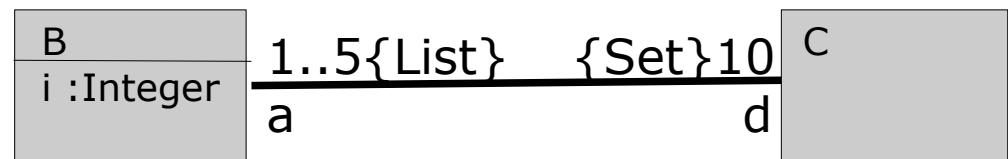
- Cardinalities in Associations can be translated canonically into MOCL invariants:



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- definition  $\text{card}_{C.a} \equiv \forall x \in C. 1 \leq |x.a| \leq 5$

# Integrity of Collection Object Attributes

- The corresponding association integrity constraints for the \*-\* -case are:



- definition  $\text{ass}_{B.d.a} \equiv \forall x \in B. x \in x.d.a$
- definition  $\text{ass}_{C.a.d} \equiv \forall x \in C. x \in x.a.d$

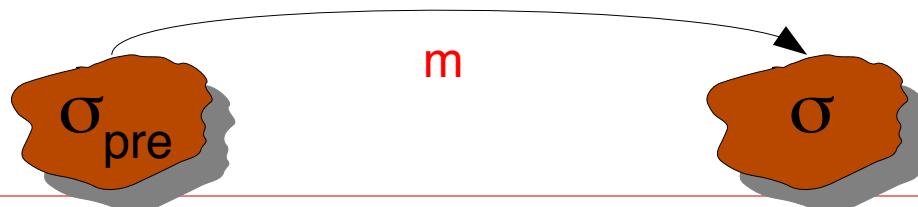
# Operations in UML and MOAL

- Many UML diagrams talk over a sequence of states (not just individual global states)

- This appears for the first time in so-called **contracts** for (Class-model) methods:

```
B
i : Integer
m(k:Integer) : Integer
```

- The « method » **m** can be seen as a « transaction » of a **B** object transforming the underlying pre-state  $\sigma_{\text{pre}}$  in the state « after » **m** yielding a post-state  $\sigma$ .

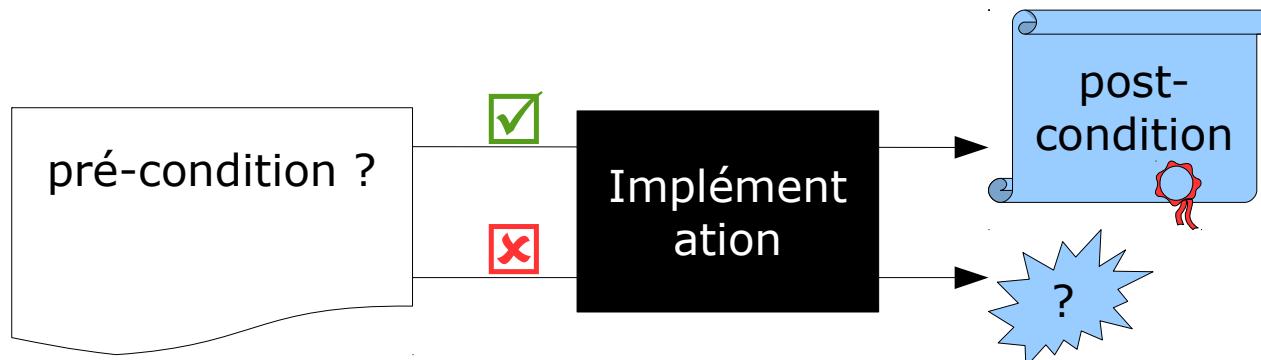


# Pré et post-conditions (piqué de Delphine ! )

Principe de la conception par contrats : contrat entre l'opération appelée et son appelant

- Appelant responsable d'assurer que la pré-condition est vraie
- Implémentation de l'opération appelée responsable d'assurer la terminaison et la post-condition à la sortie, si la pré-condition est vérifiée à l'entrée

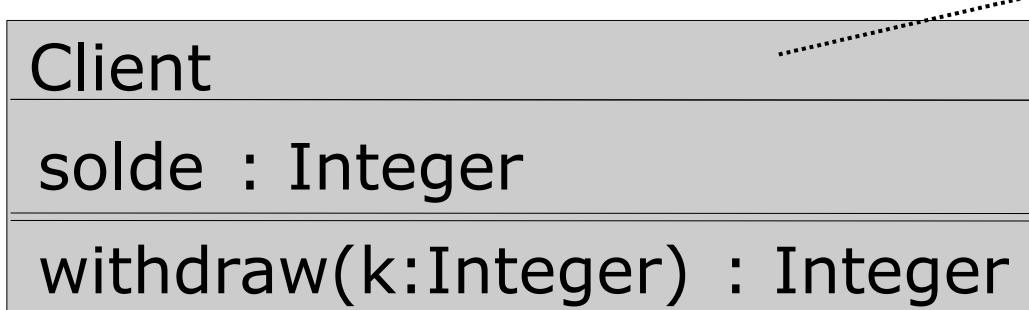
Si la pré-condition n'est pas vérifiée, aucune garantie sur l'exécution de l'opération



# Operations in UML and MOAL

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- Syntactically, contracts are annotated like this (JML-ish):

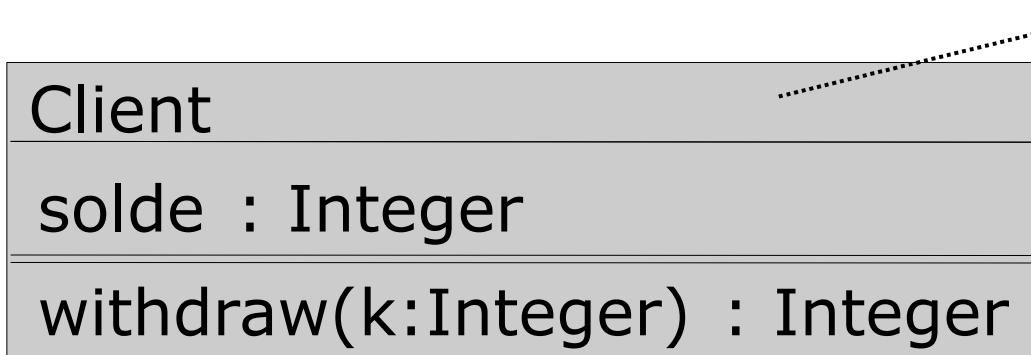


withdraw operation:  
pre:  $\text{old}(b.\text{solde}) - k \geq 0$   
post:  $b.\text{solde} = \text{old}(b.\text{solde}) - k$

# Operations in UML and MOAL

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- ... or like this (OCL-ish):



```
context c.withdraw(k):
  pre: b.solde@pre - k >= 0
  post: b.solde = b.solde@pre - k
```

# Operations in UML and MOAL Contracts

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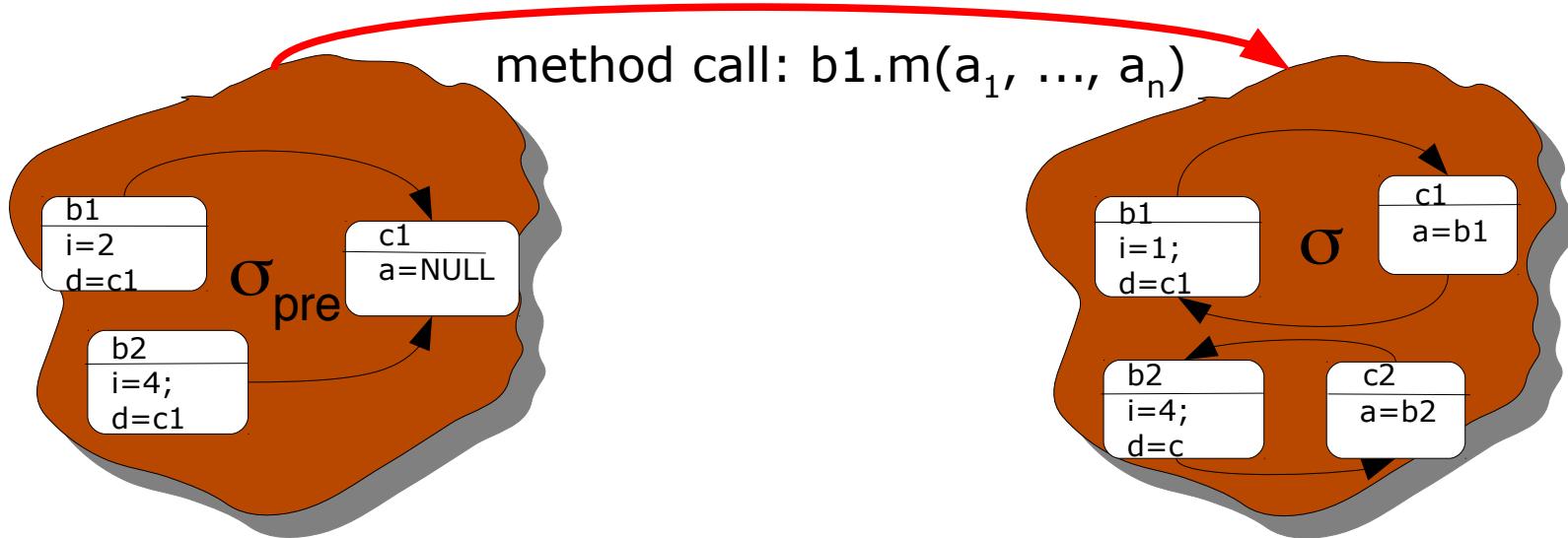
- This appears for the first time in so-called **contracts** for (Class-model) methods:

|                          |
|--------------------------|
| B                        |
| i : Integer              |
| add(k:Integer) : Integer |

- The « method » **add** can be seen as a « transaction » of a B object transforming the underlying pre-state  $\sigma_{\text{pre}}$  in the state « after » **add** yielding a post-state  $\sigma$ .

# Syntax and Semantics of MOAL Contracts

- Again: This is the view of a transaction (like in a data-base), it completely abstracts away intermediate states or time. (This possible in other models/calculi, like the Hoare-calculus, though).



# Syntax and Semantics of MOAL Contracts

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- Consequence:
  - The pre-condition is a formula referring to the  $\sigma_{\text{pre}}$  and the method arguments  $b_1, a_1, \dots, a_n$  only.
  - the post-condition is only assured if the pre-condition is satisfied
  - otherwise the method
    - ...may do anything on the state and the result, may even behave correctly , may non-terminate !
    - raise an exception  
(recommended in Java Programmer Guides for public methods to increase robustness)

# Syntax and Semantics of MOAL Contracts

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## □ Consequence:

- The post-condition is a formula referring to both  $\sigma_{\text{pre}}$  and  $\sigma$ , the method arguments  $b_1, a_1, \dots, a_n$  and the return value captured by the variable result.
- any transition is permitted that satisfies the post-condition (provided that the pre-condition is true)

# Syntax and Semantics of MOAL Contracts

## Consequence:

- The semantics of a method call:

$b1.m(a_1, \dots, a_n)$

is thus:

$$\begin{array}{c} \text{pre}_m(b1, a_1, \dots, a_n) (\sigma_{\text{pre}}) \\ \xrightarrow{\hspace{1cm}} \\ \text{post}_m(b1, a_1, \dots, a_n, \text{result})(\sigma_{\text{pre}}, \sigma) \end{array}$$

- Note that moreover all global class invariants have to be added for both pre-state  $\sigma_{\text{pre}}$  and post-state  $\sigma$  !  
For a successful transition, the following must hold:

$$\text{Inv}(\sigma_{\text{pre}}) \wedge \text{pre}_m \dots (\sigma_{\text{pre}}) \wedge \text{post} \dots (\sigma_{\text{pre}}, \sigma) \wedge \text{Inv}(\sigma)$$

# Syntax and Semantics of MOAL Contracts

## Example:

Client

`solde : Integer`

`withdraw(k:Integer) : {ok,nok}`

class invariant:  
 $c.solde \geq 0$  for all clients  $c$ .

operation  $c.withdraw(k)$  :  
pre:  $k \geq 0 \wedge old(c.solde) - k \geq 0$   
post:  $c.solde = old(c.solde) - k$   
 $\wedge result = ok$

- definition  $inv_{Client}(\sigma) \equiv \forall c \in Client(\sigma). 0 \leq c.solde(\sigma)$
- definition  $pre_{withdraw}(c, k)(\sigma) \equiv c \in Client(\sigma) \wedge 0 \leq k \wedge 0 \leq c.solde(\sigma) - k$
- definition  $post_{withdraw}(c, k, result)(\sigma_{pre}, \sigma) \equiv c \in Client(\sigma_{pre}) \wedge result = ok \wedge c.solde(\sigma) = c.solde(\sigma_{pre}) - k$

# Syntax and Semantics of MOAL Contracts

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## ❑ Notation:

- In order to relax notation, we will use for applications to  $\sigma_{\text{pre}}$  the old-notation:

$\text{Client}(\sigma_{\text{pre}})$       becomes       $\text{old}(\text{Client})$

$c.\text{solde}(\sigma_{\text{pre}})$       becomes       $\text{old}(c.\text{solde})$

etc.

# Syntax and Semantics of MOAL Contracts

## Example (revised):

Client

solde : Integer

withdraw(k:Integer) : {ok,nok}

class invariant:  
c.solde  $\geq 0$  for all clients c.

operation c.withdraw(k) :  
pre:  $k \geq 0 \wedge \text{old}(c.\text{solde}) - k \geq 0$   
post:  $c.\text{solde} = \text{old}(c.\text{solde}) - k$   
 $\wedge \text{result} = \text{ok}$

- definition  $\text{inv}_{\text{Client}} \equiv \forall c \in \text{Client}. \quad 0 \leq c.\text{solde}$
- definition  $\text{pre}_{\text{withdraw}}(c, k) \equiv$   
 $c \in \text{Client} \wedge 0 \leq k \wedge 0 \leq c.\text{solde} - k$
- definition  $\text{post}_{\text{withdraw}}(c, k, \text{result}) \equiv$   
~~MOAL convention~~  
 $c \in \text{old}(\text{Client}) \wedge \text{result} = \text{ok}$   
 $c.\text{solde} = \text{old}(c.\text{solde}) - k \wedge$

# Syntax and Semantics of MOAL Contracts

## ❑ Alternative Example:

Client

solde : Integer

withdraw(k:Integer) : {ok,nok}

class invariant:  
c.solde  $\geq 0$  for all clients c.

operation c.withdraw(k) :  
pre: true  
post:  
if  $k \geq 0 \wedge \text{old}(c.\text{solde}) - k \geq 0$   
then  $c.\text{solde} = \text{old}(c.\text{solde}) - k$   
 $\wedge \text{result} = \text{ok}$   
else  $\text{result} = \text{nok}$

What are the differences  
between these contracts?

# Semantics of MOAL Contracts

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- Two predicates are helpful when defining contracts. They exceptionally refer to both  $(\sigma_{\text{pre}}, \sigma)$ 
  - $\text{isNew}(p)(\sigma_{\text{pre}}, \sigma)$  is true only if object p of class C does not exist in  $\sigma_{\text{pre}}$  but exists in  $\sigma$
  - $\text{modifiesOnly}(S)(\sigma_{\text{pre}}, \sigma)$  is only true iff
    - all objects in  $\sigma_{\text{pre}}$  are **except those in S** identical in  $\sigma$
    - all objects exist either in  $\sigma$  are or are contained in S

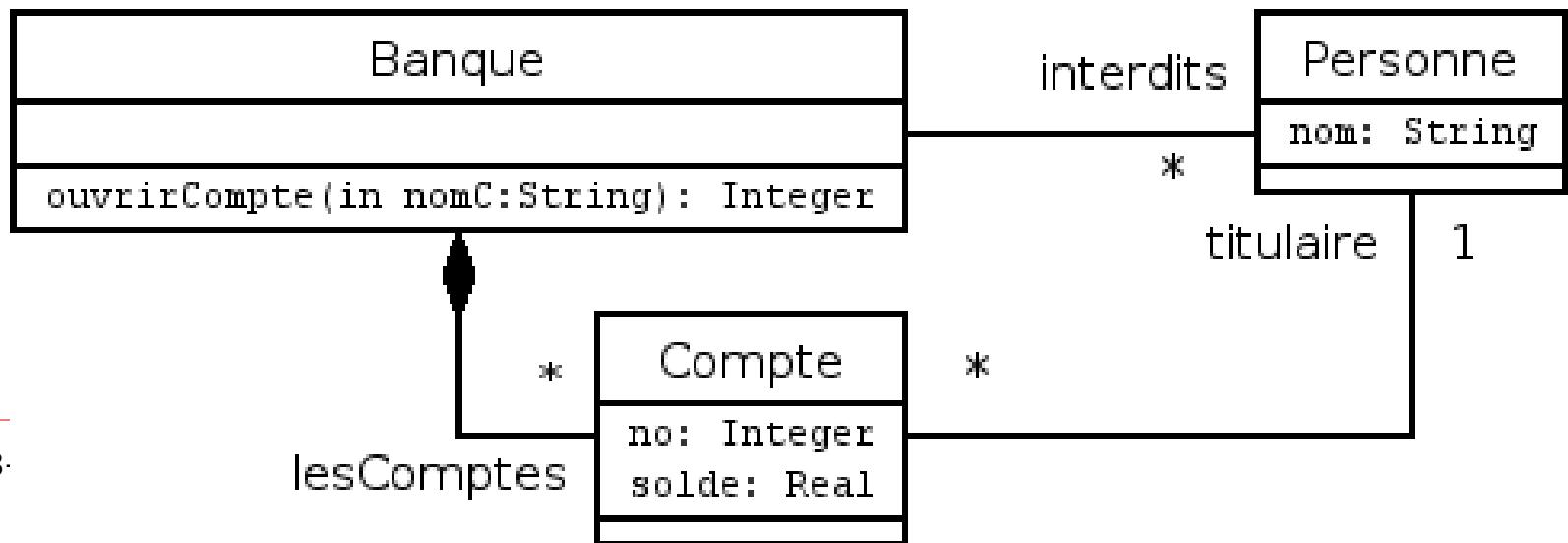
With this predicate, one can express : „and nothing else changes“. It is also called «framing condition»

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# A Revision of the Example: Bank

Opening a bank account. Constraints:

- ❑ there is a blacklist
- ❑ no more overdraft than 200 EUR
- ❑ there is a present of 15 euros in the initial account
- ❑ account numbers must be distinct.



# A Revision of the Example: Bank (2)

- ```

definition preouvrirCompte(b:Banque, nomC:String)≡
     $\forall p \in \text{Personne}. p.\text{nom} \neq \text{nomC}$ 

definition postouvrirCompte(b:Banque, nomC:String, r:Integer)≡
    | {p ∈ Personne | p.nom = nomC} | = 1
     $\wedge \forall p \in \text{Personne}. p.\text{nom} = \text{nomC} \rightarrow \text{isNew}(p)$ 
     $\wedge | \{c \in \text{Compte} | c.\text{titulaire.nom} = \text{nomC}\} | = 1$ 
     $\wedge \forall c \in \text{Compte}. c.\text{titulaire.nom} = \text{nomC} \rightarrow c.\text{solde} = 15$ 
         $\wedge \text{isNew}(c)$ 
     $\wedge b.\text{lesComptes} = \text{old}(b.\text{lesComptes}) \cup$ 
        {c ∈ Compte | c.titulaire.nom = nomC}
     $\wedge b.\text{interdits} = \text{old}(b.\text{interdits}) \cup$ 
        {c ∈ Compte | c.titulaire.nom = nomC}
     $\wedge \text{modifiesOnly}(\{b\} \cup \{c \in \text{Compte} | c.\text{titulaire.nom} = \text{nomC}\}$ 
         $\cup \{p \in \text{Personne} | p.\text{nom} = \text{nomC}\})$ 

```

# Operations in UML and MOAL

- Example:

Client

solde : Integer

deposit(k:Integer) : {ok,nok}

withdraw(k:Integer) : {ok,nok}

solde() : Integer

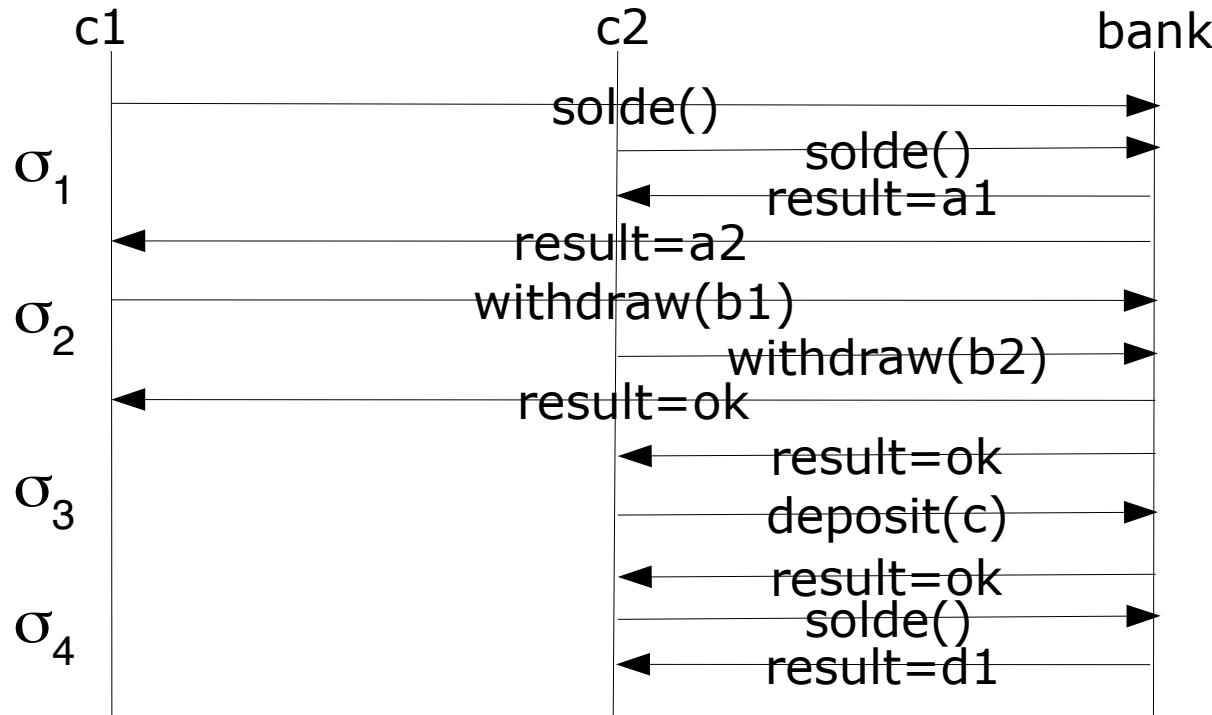
deposit operation:  
pre:  $k \geq 0$   
post:  $b.solde = \text{old}(b.solde) + k$

withdraw operation:  
pre:  $\text{old}(b.solde) - k \geq 0$   
post:  $b.solde = \text{old}(b.solde) - k$   
post: result = ok

solde query:  
post: result =  $\text{old}(b.solde)$

# Operations in UML and MOAL

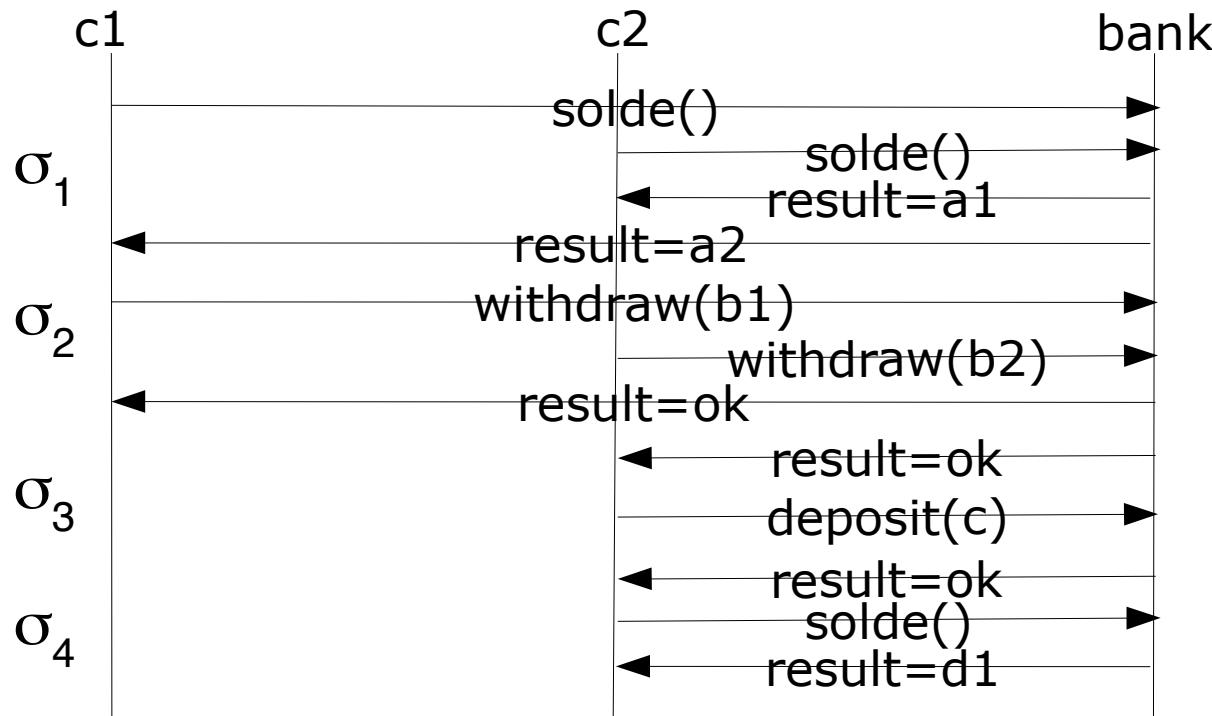
- Abstract Concurrent Test Scenario:



assert  $c1.\text{solde}(\sigma_4)=a2-b1 \wedge b1 \geq 0 \wedge a2 \geq b1$

# Operations in UML and MOAL

## □ Abstract Concurrent Test Scenario:



Any instance of **b1** and **a1** is a test ! This is a „Test Schema“ !  
Note: **b1** can be chosen dynamically during the test !

# Summary

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- MOAL makes the UML to a real, formal specification language
- MOAL can be used to annotate Class Models, Sequence Diagrams and State Machines
- Working out, making explicit the constraints of these Diagrams is an important technique in the transition from Analysis documents to Designs.