Interactive Theorem Proving and Applications

Année 2020-21

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TP 5 - Inductive Constructs for Modeling Regular Languages

Semaine du 1 fevrier 2021

Objective : Defining a denotational semantics for as more-or-less standard regular expression language. Following common terminology in automata theory, we will call a list of atoms a *word*, and a set of words a *language*. The *denotational semantics* we are referring here is a function that maps an (abstract) *syntax* to a set of *deonotations*, i.e. regular expressions to the language they denote.

Exercice 1 (Inductive sets - Inductive Proofs)
Define a (polymorphic) regular expression language `'a rexp` with the alternatives :

- `Empty` (denoted `<>`)
- `Atom` (a singleton, denoted $\lfloor\_\rfloor$)
- `Alt` (for alternative, denoted _|_)
- `Conc` (for sequence, denoted _ : _)
- `Star` (for arbitrary repetition)

Tasks :

1. Why is $((A ::' arexp)|B) = (B|A)$ not true in general ?
2. Define inductively : if A is a language, then $star\ A$ is the set of all repetitions over A.
3. Define recursively L, the language of a regular expression.
4. Prove $star\{\} = \{[]\}$ and therefore $star(star\{[]\}) = \{[]\}$.
5. Prove that L commutes over _|_.
6. Prove that under L, _ : _ distributes over _|_ (left and right).
7. Prove that the word `''acbc''` is in the language of $Star((\lfloor CHR''a''\rfloor||\lfloor CHR''b''\rfloor) : \lfloor CHR''c''\rfloor)$

Note : Main provides the notation `CHR ''a''` for "the character a". Strings are defined as lists of characters.

Exercice 2 (OPTIONAL : Report)
(IN CASE THAT YOU WANT TO HAVE IT GRADED. RECALL THAT 2 OUT OF 6 TP's SHOULD BE SUBMITTED.)

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 3 pages max (except screenshots and other figures).