L3 Mention Informatique
Parcours Informatique et MIAGE

Génie Logiciel Avancé - Advanced Software Engineering

Black-Box Tests

Burkhart Wolff
wolff@lri.fr

Towards Static Specification-based Unit Test

- How can we test during development
  (at coding time, even at design-time ?)
- How can we test "systematically"?
  - What could be a test-generation method?
  - What could be an algorithm to generate tests?
  - What could be a coverage criterion ?
    (or: adequacy criterion,
    telling that we "tested enough")

Difficulties with Static Unit Tests so far

- Some empirical observations:
  - No relation between detection order and detection difficulty
  - No relation between detection difficulty and correction
  - The more errors you found, the more you find more…
  - The quality of a test set is independent of its size.

Functional Unit Test : An Example

The specification in UML/MOAL:

```
Triangles
-----------------------------------------
a, b, c: Integer
=========================================
- mk(Integer,Integer,Integer):Triangle
- is_Triangle(): {equ (*equilateral*),
                 iso (*isosceles*),
                 arb (*arbitrary*)}
```

Functional Unit Test : An Example

Recall:

inv 0<a ∧ 0<b ∧ 0<c
inv c≤a+b ∧ a≤b+c ∧ b≤c+a

Triangles

```
a, b, c: Integer
```

```
- mk(Integer,Integer,Integer):Triangle
- is_Triangle(): {equ (*equilateral*),
                  iso (*isosceles*),
                  arb (*arbitrary*)}
```

operation t.is_Triangle():

pre t ≠ null

post t.a=t.b ∧ t.b=t.c ⟶ result=equ

post (t.a≠t.b ∨ t.b≠t.c ∨ t.a≠t.c) ∧ (t.a=t.b ∨ t.b=t.c ∨ t.a=t.c)) ⟶ result=iso

post (t.a≠t.b ∧ t.b≠t.c ∧ t.a≠t.c)) ⟶ result=arb

post modifiesOnly({})

Generating Test-Data by Example

- Consider the test specification (the "Test Goal"):

  mk(x,y,z).isTriangle() ≡ X

  i.e. for which input (x,y,z) should an implementation of our contract yield which X ?

  Note that we define mk(0,0,0) to invalid, as well as all other invalid triangles ...

Recall : Intuitive Test-Data Generation

- an arbitrary valid triangle: (3, 4, 5)
- an equilateral triangle: (5, 5, 5)
- an isoscele triangle and its permutations :

  (6, 6, 7), (7, 6, 6), (6, 7, 6)
- impossible triangles and their permutations :

  (1, 2, 4), (4, 1, 2), (2, 4, 1) -- $x + y > z$

  (1, 2, 3), (2, 4, 2), (5, 3, 2) -- $x + y = z$ (necessary?)
- a zero length : (0, 5, 4), (4, 0, 5),
- . . .
- Would we have to consider negative values?

Test-Data Generation

- Ouf, is there a systematic and automatic way to compute
  all these cases ?

  Well, lets see and calculate ...

Test-Data Generation

- Recall the test specification:

  mk(x,y,z).isTriangle() = r

Test-Data Generation

- Recall the test specification:

  mk(x,y,z).isTriangle() = r

$\equiv$ $inv_{Triangle}(\sigma) \wedge pre_{isTriangle}(mk(x,y,z))(\sigma) \wedge$
$inv_{Triangle}(\sigma') \wedge post_{isTriangle}(mk(x,y,z),r)(\sigma,\sigma')$

(* see semantics in MOAL II, page 22. *)

Some Facts:

- From modifiesOnly({}) follows $\sigma = \sigma'$ hence

  $inv_{Triangle}(\sigma) = inv_{Triangle}(\sigma')$

- From mk(x,y,z) $\neq$ null (see $pre_{isTriangle}$) and from $inv_{Triangle}(\sigma)$ and mk(x,y,z) $\in$ Triangle (σ) follows that:

  $0<x \wedge 0<y \wedge 0<z \wedge x \leq y+z \wedge y \leq x+z \wedge z \leq x+y$ ($\equiv$ inv)

Revision: Boolean Logic + Some Basic Rules

- $\neg(a \land b) = \neg a \lor \neg b$ (* deMorgan1 *)
- $\neg(a \lor b) = \neg a \land \neg b$ (* deMorgan2 *)
- $a \land (b \lor c) = (a \land b) \lor (a \land c)$
- $\neg(\neg a) = a$, $a \lor \neg a = T$, , $a \land \neg a = F$,
- $a \land b = b \land a$; $a \lor b = b \lor a$
- $a \land (b \land c) = (a \land b) \land c$
- $a \lor (b \lor c) = (a \lor b) \lor c$
- $a \longrightarrow b = (\neg a) \lor b$
- $(a=b \land P(a)) = P(b)$ (* one point rule *)

- let x = E in C(x) = C(E) (* let elimination *)
- if c then C else D = $(c \land C) \lor (\neg c \land D) = (c \longrightarrow C) \land (\neg c \longrightarrow D)$

Test-Data Generation

- Recall the test specification:

  mk(x,y,z).isTriangle() = r

$\equiv$ $inv_{Triangle}(\sigma) \wedge pre_{isTriangle}(mk(x,y,z))(\sigma) \wedge$

$inv_{Triangle}(\sigma') \wedge post_{isTriangle}(mk(x,y,z),r)(\sigma,\sigma')$

(* see semantics d'un appel de methopde, in MOAL II, page 22. *)

Some Facts:

- $arb \neq equ \neq iso$
- $post_{isTriangle}(mk(x,y,z),r)(\sigma,\sigma)$ can be simplified to:

  $(x=y \wedge y=z \longrightarrow r=equ) \wedge$

  $((x\neq y \vee y\neq z) \wedge (x=y \vee y=z \vee x=z) \longrightarrow r=iso) \wedge$

  $((x\neq y \wedge y\neq z \wedge x\neq z) \longrightarrow r=arb)$

Test-Data Generation

- Summing up:

$$mk(x,y,z).isTriangle() = r$$

$$\equiv \; inv_{Triangle}(\sigma) \wedge pre_{isTriangle}(mk(x,y,z))(\sigma) \wedge$$
$$inv_{Triangle}(\sigma') \wedge post_{isTriangle}(mk(x,y,z),r)(\sigma,\sigma')$$

$\Longrightarrow$ (* the discussed facts *)

$$inv \wedge$$
$$(x=y \wedge y=z \longrightarrow r=equ) \wedge$$
$$((x\neq y \vee y\neq z) \wedge (x=y \vee y=z \vee x=z) \longrightarrow r=iso) \wedge$$
$$(x\neq y \wedge y\neq z \wedge x\neq z \longrightarrow r=arb)$$

Test-Data Generation

- Recall the test specification:

$$\texttt{inv} \wedge (x{=}y \wedge y{=}z \longrightarrow r{=}equ) \wedge$$
$$((x{\neq}y \vee y{\neq}z) \wedge (x{=}y \vee y{=}z \vee x{=}z) \longrightarrow r{=}iso) \wedge$$
$$(x{\neq}y \wedge y{\neq}z \wedge x{\neq}z \longrightarrow r{=}arb)$$

≡ (* elimination ⟶ , deMorgan*)

$$\texttt{inv} \wedge$$
$$(x{\neq}y \vee y{\neq}z \vee r{=}equ) \wedge$$
$$((x{=}y \wedge y{=}z) \vee (x{\neq}y \wedge y{\neq}z \wedge x{\neq}z) \vee r{=}iso) \wedge$$
$$(x{=}y \vee y{=}z \vee x{=}z \vee r{=}arb)$$

Test-Data Generation

- This first part of the calculation could be called

  PURIFICATION

  We eliminate UML, object-orientation, MOAL etcpp and reduce it to the pure logical core ...

  Now, under which precise conditions do we have

  - r = iso
  - r = arb
  - r = equ ???

Test-Data Generation

- This first part of the calculation could be called

  PURIFICATION

  We eliminate UML, object-orientation, MOAL etcpp and reduce it to the pure logical core ...

  Can we transform the spec into the form

  - $A_1 \wedge \ldots \wedge A_i \wedge r = iso$
  - $B_1 \wedge \ldots \wedge B_k \wedge r = arb$
  - $C_1 \wedge \ldots \wedge C_l \wedge r = equ$???

Test-Data Generation

- This first part of the calculation could be called

  PURIFICATION

  We eliminate UML, object-orientation, MOAL etcpp and reduce it to the pure logical core ...

  Can we transform the spec into a

  Disjunctive Normal Form (DNF) ?

Excursion

- Generalized Distribution Laws:

$(A_1 \vee A_2) \wedge (B_1 \vee B_2) = (A_1 \wedge (B_1 \vee B_2)) \vee (A_2 \wedge (B_1 \vee B_2))$

$= (A_1 \wedge B_1) \vee (A_2 \wedge B_1) \vee (A_1 \wedge B_2) \vee (A_2 \wedge B_2)$

$(A_1 \vee A_2 \vee A_3) \wedge (B_1 \vee B_2 \vee B_3) \wedge (C_1 \vee C_2 \vee C_3)$

$= \ldots$

$= (A_1 \wedge B_1 \wedge C_1) \vee (A_1 \wedge B_1 \wedge C_2) \vee (A_1 \wedge B_1 \wedge C_3) \vee$

$(A_2 \wedge B_1 \wedge C_1) \vee (A_2 \wedge B_1 \wedge C_2) \vee (A_2 \wedge B_1 \wedge C_3) \vee$

…

$(A_1 \wedge B_3 \wedge C_3) \vee (A_2 \wedge B_3 \wedge C_3) \vee (A_3 \wedge B_3 \wedge C_3)$

Test-Data Generation

- Recall the test specification:

  ...

  $\equiv$ inv $\wedge$

  $(x \neq y \vee y \neq z \vee r=equ) \wedge$

  $(x=y \vee y=z \vee x=z \vee r=arb) \wedge$

  $((x=y \wedge y=z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r=iso)$

$\equiv$ (* generalized distribution 2nd/3rd line *)

inv $\wedge$

$((x \neq y \wedge x=y) \vee (x \neq y \wedge y=z) \vee (x \neq y \wedge x=z) \vee (x \neq y \wedge r=arb)) \vee$

$((y \neq z \wedge x=y) \vee (y \neq z \wedge y=z) \vee (y \neq z \wedge x=z) \vee (y \neq z \wedge r=arb)) \vee$

$((r=equ \wedge x=y) \vee (r=equ \wedge y=z) \vee (r=equ \wedge x=z) \vee (r=equ \wedge r=arb)) \vee$

$((x=y \wedge y=z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r=iso)$

Test-Data Generation

- Recall the test specification:

$$\ldots$$

$$\equiv \; inv \wedge$$
$$(x \neq y \vee y \neq z \vee r = equ) \wedge$$
$$(x = y \vee y = z \vee x = z \vee r = arb) \wedge$$
$$((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$$

≡ (* elimination contradictions *)

$$inv \wedge$$
$$((x \neq y \wedge x = y) \vee (x \neq y \wedge y = z) \vee (x \neq y \wedge x = z) \vee (x \neq y \wedge r = arb) \vee$$
$$(y \neq z \wedge x = y) \vee (y \neq z \wedge y = z) \vee (y \neq z \wedge x = z) \vee (y \neq z \wedge r = arb) \vee$$
$$(r = equ \wedge x = y) \vee (r = equ \wedge y = z) \vee (r = equ \wedge x = z) \vee (r = equ \wedge r = arb)) \vee$$
$$((x = y \wedge y = z) \vee (x \neq y \wedge y \neq z \wedge x \neq z) \vee r = iso)$$

Test-Data Generation

- Recall the test specification:

  ...

  ≡ (* elimination contradictions *)

```
inv ∧
   ((x≠y ∧ y=z)∨(x≠y ∧ x=z)∨(x≠y ∧ r=arb) ∨
    (y≠z ∧ x=y)∨(y≠z ∧ x=z)∨(y≠z ∧ r=arb) ∨
    (r=equ∧x=y)∨(r=equ∧y=z)∨(r=equ∧x=z)) ∧
   ((x=y ∧ y=z) ∨ (x≠y ∧ y≠z ∧ x≠z) ∨ r=iso)
```

Test-Data Generation

- ≡ (* generalized distribution 2nd/3rd ((9 * 3 = 27 cases !)*)

```
inv ∧
   ((x≠y∧y=z∧x=y∧y=z)∨(x≠y∧x=z∧
                       x=y∧y=z)∨(x≠y∧r=arb∧x=y∧y=z) ∨
    (y≠z∧x=y∧x=y∧y=z)∨(y≠z∧x=z∧
                       x=y∧y=z)∨(y≠z∧r=arb∧x=y∧y=z) ∨
    (r=equ∧x=y∧x=y∧y=z)∨(r=equ∧
                       y=z∧x=y∧y=z)∨(r=equ∧x=z∧x=y∧y=z)) ∨
   ((x≠y∧y=z∧x≠y∧y≠z∧x≠z)∨(x≠y∧x=z∧x≠y∧y≠z∧x≠z)∨(x≠y∧r=arb
   ∧ x≠y∧y≠z∧x≠z)∨(y≠z∧x=y∧x≠y∧y≠z∧x≠z)∨(y≠z∧x=z∧x≠y∧y≠z∧
   x≠z)∨(y≠z∧r=arb∧x≠y∧y≠z∧x≠z)∨(r=equ∧x=y∧x≠y∧y≠z∧x≠z)∨(
   r=equ∧y=z∧x≠y∧y≠z∧x≠z)∨(r=equ∧x=z∧x≠y∧y≠z∧ x≠z)) ∨
   ((x≠y ∧ y=z∧r=iso)∨(x≠y ∧ x=z∧r=iso)∨(x≠y∧r=arb∧r=iso)
    ∨(y≠z∧x=y∧r=iso)∨(y≠z∧x=z∧r=iso)∨(y≠z∧r=arb∧r=iso) ∨
    (r=equ∧x=y∧r=iso)∨(r=equ∧y=z∧r=iso)∨(r=equ∧x=z∧r=iso))
```

Test-Data Generation

- ≡ (* elimination of the contradictions and redundancies *)

```
inv ∧
  ((x≠y∧y=z∧x=y∧y=z)∨(x≠y∧x=z∧
                        x=y∧y=z)∨(x≠y∧r=arb∧x=y∧y=z) ∨
   (y≠z∧x=y∧x=y∧y=z)∨(y≠z∧x=z∧
                        x=y∧y=z)∨(y≠z∧r=arb∧x=y∧y=z) ∨
   (r=equ∧x=y∧x=y∧y=z)∨(r=equ∧
                        y=z∧x=y∧y=z)∨(r=equ∧x=z∧x=y∧y=z))∨
  ((x≠y∧y=z∧x≠y∧y≠z∧x≠z)∨(x≠y∧x=z∧x≠y∧y≠z∧x≠z)∨(x≠y∧r=arb
  ∧ x≠y∧y≠z∧x≠z)∨(y≠z∧x=y∧x≠y∧y≠z∧x≠z)∨(y≠z∧x=z∧x≠y∧y≠z∧
  x≠z)∨(y≠z∧r=arb∧x≠y∧y≠z∧x≠z)∨(r=equ∧x=y∧x≠y∧y≠z∧x≠z)∨(
  r=equ∧y=z∧x≠y∧y≠z∧x≠z)∨(r=equ∧x=z∧x≠y∧y≠z∧ x≠z))∨
  ((x≠y ∧ y=z∧r=iso)∨(x≠y ∧ x=z∧r=iso)∨(x≠y∧r=arb∧r=iso)
   ∨(y≠z∧x=y∧r=iso)∨(y≠z∧x=z∧r=iso)∨(y≠z∧r=arb∧r=iso) ∨
   (r=equ∧x=y∧r=iso)∨(r=equ∧y=z∧r=iso)∨(r=equ∧x=z∧r=iso))
```

Test-Data Generation

- ≡ (* cleanup, distribution *)

```
(inv ∧ x=y ∧ x=y ∧ y=z ∧ r=equ) ∨            (1)
(inv ∧ x≠y ∧ y≠z ∧ x≠z ∧ r=arb ) ∨           (2)
(inv ∧ x≠y ∧ y=z ∧ r=iso) ∨                  (3)
(inv ∧ x≠y ∧ x=z ∧ r=iso) ∨                  (4)
(inv ∧ y≠z ∧ x=y ∧ r=iso) ∨                  (5)
(inv ∧ y≠z ∧ x=z ∧ r=iso)                    (6)
```

- Test-Case-Construction by DNF Method

  yields six abstract test cases

  relating input x y z to output r

- Note: In general, output r is not necessarily uniquely defined as in our example ...
  The spec can be non-deterministic admitting several results.

Test-Data Generation

- Test-Data-Selection:
  For each abstract test-case, we construct one concrete test, by choosing values that make the abstract test case true (« that satisfies the abstract test case »)

case	x	y	z	result
(1)	3	3	3	equ
(2)	3	4	6	arb
(3)	4	5	5	iso
(4)	5	4	5	iso
(5)	5	5	4	iso
(6)	4	3	4	iso

Test-Data Generation

- Intuitively, what does it mean that we "covered" the DNF by tests
  - Any basic predicate ("literal") has been used at least one time
    - … provided it is not contradictory ("A=False")
    - … provided that it is not redundant ("A=True")
    - … provided it is not implied by another literal, i.e. it is subsumed ("B $\longrightarrow$ A")

Test-Data Generation

- A First Summary on the Test-Generation Method:
  - PHASE I: Stripping the Domain-Language (UML-MOAL) away, "purification"
  - PHASE II: Abstract Test Case Construction by "DNF computation"
  - PHASE III: Constraint Resolution (by solvers like CVC4 or Z3) "Test Data Selection"
  - COVERAGE CRITERION:
    DNF - coverage of the Spec; for each abstract test-case one concrete test-input is constructed.
    (**ISO/IEC/IEEE** 29119 calls this: Equivalence class testing)
- Remark: During Codiung phase, when the Spec does not change, the test-data-selection can be repeated easily creating always different test sets …

Test-Data Generation

- Variants:
  - Alternative to PHASE II (DNF construction): Predicate Abstraction and Tableaux-Exploration.

    Reconsider the (purified) specification:

    $$\begin{array}{l} \mathrm{inv} \wedge \\ (x=y \wedge y=z \longrightarrow r=equ) \wedge \\ ((x\neq y \vee y\neq z) \wedge (x=y \vee y=z \vee x=z) \longrightarrow r=iso) \wedge \\ (x\neq y \wedge y\neq z \wedge x\neq z \longrightarrow r=arb) \end{array}$$

    It is possible to abstract this spec to a fairly small number of „base predicates" ... They should be logically independent and not contain the output variable...

Test-Data Generation

- Variants:
  - Alternative to PHASE II (DNF construction): Predicate Abstraction and Tableaux-Exploration.

    Reconsider the (purified) specification:

    $$\begin{array}{l} \mathtt{inv} \wedge \\ (A \wedge B \longrightarrow r{=}equ) \wedge \\ ((\neg A \vee \neg B) \wedge (A \vee B \vee C) \longrightarrow r{=}iso) \wedge \\ (\neg A \wedge \neg B \wedge \neg C \longrightarrow r{=}arb) \end{array}$$

    where $A \mapsto x{=}y$, $B \mapsto y{=}z$, $C \mapsto x{=}z$

    (actually: A and B imply C)

Test-Data Generation

- Variants:
  - ... Now we can construct a tableau and get by simplification:

case	A	B	C	spec reduces to
(1)	T	T	T	• `r=equ`
(2)	T	T	F	• `r=equ  (!!!)`
(3)	T	F	T	• `r=iso`
(4)	T	F	F	• `r=iso`
(5)	F	T	T	• `r=iso`
(6)	F	T	F	• `r=iso`
(7)	F	F	T	• `r=iso`
(8)	F	F	F	• `r=arb`

Test-Data Generation

- Variants:
  - PHASE III: Borderline analysis.
    Principle: we replace in our DNF inequalities by „the closest values that make the spec true“

$$x \neq y \quad \mapsto \quad x = y + 1 \lor x = y - 1$$

$$x \leq y \quad \mapsto \quad x = y \lor x < y$$

$$x < y \quad \mapsto \quad x = y - 1 \qquad \text{etc.}$$

  - … and recompute the DNF. In general, this gives a much finer mesh …

Test-Data Generation

- Variants:
  - PHASE I: Test for exceptional behaviour.

    We negate the precondition and to DNF generation on the precondition only.

    Test objectives could be:

    - should raise an exception if public
    - should not diverge

Test-Data Generation

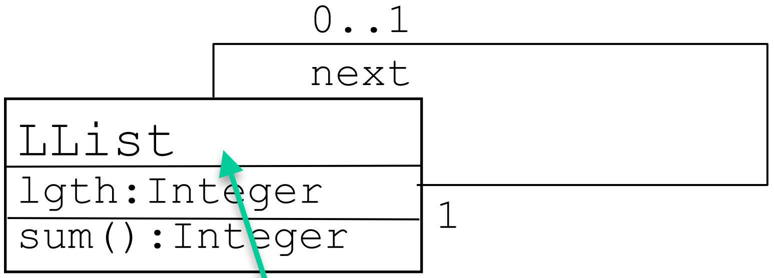
- How to handle Recursion ?

Test-Data Generation

- How to handle Recursion ?

  In UML/MOAL, recursion occurs (at least) at two points:

  - at the level of data



invariant:

inv_{LList} ≡ ∀node∈LList.
```
node.lgth =if node.next = null
           then 1
           else next.lgth + 1
```

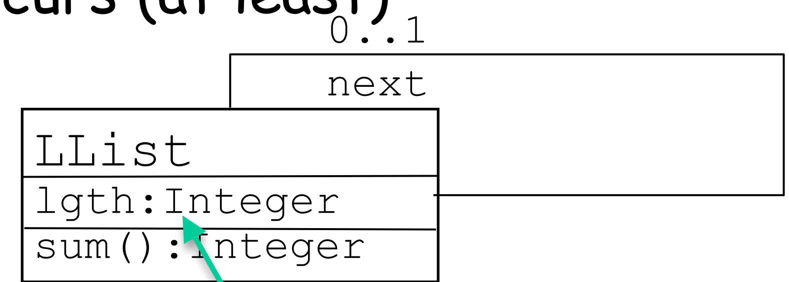
Note that this excludes cyclic lists !!!

Test-Data Generation

- How to handle Recursion ?

  In UML/MOAL, recursion occurs (at least) at two points:

  - at the level of operations (post-conds may contain calls ...)



query contract (modifiesOnly({})):

definition pre_{sum}(l) ≡ True

definition $post_{sum}$(l,res)≡ res=if l.next=null then l.lgth
else l.lgth + l.next.sum()

definition sum(l)≡ arb{r|pre_{sum}(l) ∧ $post_{sum}$(l,r)}

Note that arb(S) gives an arbitrary member of S: arb(S)∈S. Since from x=arb({y}) follows x=y; thus sum(l) is (uniquely) defined.

Test-Data Generation

- Prerequisite: We present the invariant as recursive predicate.

definition inv_{LList_Core} `n σ ≡(n.lgth(σ) = if n.next(σ)=null then 1`
`else n.next.lgth(σ) + 1)`

we have:

inv_{LList} `(σ) = ∀n∈LList(σ).` inv_{LList_Core} `n σ`

and

inv_{LList_Core} `(n)(σ)= (if n.next(σ)=null then n.lgth(σ) = 1`
`else n.lgth(σ) =n.next.lgth(σ) + 1`
`∧ n.next(σ)∈LList(σ)`
`∧` inv_{LList_Core} `(n.next)(σ))`

Furthermore we have:

```
sum(l)(σ',σ) = if l.next(σ)=null then l.lgth(σ)
                 else l.lgth(σ) + sum(l.next)(σ',σ)
```

We have $\sigma'=\sigma$ (why?). We will again apply (σ',σ) - convention.

Test-Data Generation

- Consider the test specification:

$X.sum() \equiv Y$ (for some $X \in LList$, i.e. $X \neq null$)

$\equiv inv_{LList}(X) \wedge pre_{sum}(X) \wedge post_{sum}(X,Y)$

where:

$pre_{sum}(X) \equiv true$

$post_{sum}(X,Y) \equiv (\text{if } X.next = null \text{ then } Y = X.lgth$
$\text{else } Y = X.lgth + sum(X.next))$
$\equiv (X.next=null \wedge Y = X.lgth)$
$\vee (X.next \neq null \wedge Y = X.lgth+sum(X.next)$

Test-Data Generation

- DNF computation yields already the test cases:

X.sum() ≡ Y (for some X∈LList, i.e. X≠null)

$\Longrightarrow \texttt{inv}_{\texttt{LList\_Core}}(\texttt{X}) \wedge \texttt{post}_{\texttt{sum}}(\texttt{X,Y}))$

≡ (if X.next=null then X.lgth = 1

else X.lgth =X.next.lgth+1 ∧ X.next∈LList ∧ $\text{inv}_{\text{LList\_Core}}$(X.next)) ∧

(if X.next = null then Y = X.lgth

else Y = X.lgth + sum(X.next))

≡ (if c then C else D elim, DNF)

(X.next=null ∧ X.lgth=1 ∧ Y = X.lgth)

∨ (X.next≠null ∧ X.lgth =X.next.lgth+1

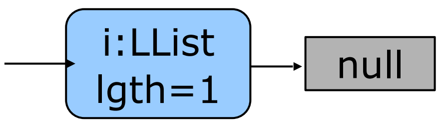
∧ X.next∈LList ∧ $\text{inv}_{\text{LList\_Core}}$(X.next)

∧ Y = X.lgth+sum(X.next))

New Test-Case!!

Test-Data Generation

- Intermediate Summary: test-cases known so far ?

X	Y
	1
...	...
...	...

Test-Data Generation

- Prerequisite: We present the invariant as recursive predicate.

inv_{LList_Core}(n)= (if n.next=null then n.lgth = 1
else n.lgth =n.next.lgth + 1
∧ n.next∈LList ∧ inv_{LList_Core}(n.next))

- ```
  sum(l) = if l.next=null then l.lgth
           else l.lgth + sum(l.next)
  ```

```
sum(l) = if X.next.next=null then X.next.lgth
          else X.next.lgth + sum(X.next.next)
```
```

Test-Data Generation

- DNF computation yields already the test cases:

  X.sum() ≡ Y (for some $X \in$ LList, i.e. $X \neq$ null)

  ⟹ … ≡ …

  ≡ (unfolding sum and inv_{LList_Core})

```
(X.next=null ∧ X.lgth=1 ∧ Y = X.lgth)
  ∨ (X.next≠null ∧ X.lgth=X.next.lgth+1 ∧ X.next∈LList
    ∧ (if X.next.next=null then X.next.lgth = 1
            else X.next.lgth =X.next.next.lgth + 1
                 ∧ X.next.next∈LList ∧ inv_LList_Core(X.next.next))
    ∧ (Y = X.lgth+(if X.next.next=null then X.next.lgth
                      else X.next.lgth + sum(X.next.next)))
```

Test-Data Generation

- DNF computation yields already the test cases:

$X.sum() \equiv Y$ (for some $X \in LList$, i.e. $X \neq null$)

$\Longrightarrow$ ... ≣ ...

≣ (DNF partial)

```
(X.next=null ∧ X.lgth=1 ∧ Y = X.lgth)
 ∨ (X.next≠null ∧ X.lgth=X.next.lgth+1 ∧ X.next∈LList
   ∧ ( (X.next.next=null ∧ X.next.lgth = 1 ∧ Y = X.lgth+X.next.lgth)
     ∨ (X.next.next≠null ∧ X.next.lgth=X.next.next.lgth+1
       ∧ X.next.next∈LList ∧ inv_LList_Core(X.next.next)
       ∧ Y = X.lgth+ X.next.lgth + sum(X.next.next))
     )
```

Test-Data Generation

- DNF computation yields already the test cases:

X.sum() ≡ Y (for some X∈LList, i.e. X≠null)

⟹ ... ≡ ...

≡ (DNF partial)

```
(X.next=null ∧ X.lgth=1 ∧ Y = X.lgth)
  ∨ (X.next≠null ∧ X.lgth=X.next.lgth+1 ∧ X.next∈LList
     ∧ X.next.next=null ∧ X.next.lgth=1 ∧ Y = X.lgth+X.next.lgth))
  ∨ (X.next≠null ∧ X.lgth=X.next.lgth+1 ∧ X.next∈LList
     ∧ X.next.next≠null ∧ X.next.lgth=X.next.next.lgth+1
           ∧ X.next.next∈LList ∧ inv_LList_Core(X.next.next)
           ∧ Y = X.lgth+ X.next.lgth + sum(X.next.next))
```

New Test-Case!!

Test-Data Generation

- Intermediate Summary: test-cases known so far ?

X	Y
→ i:LList lgth=1 → null	1
→ i:LList lgth=2 → i:LList lgth=1 → null	2
...	...

Summary: Symbolic Test-Case Generation

- ... and we could continue forever
  - compile to semantics

    (-> convert in mathematical, logical notation)
  - use recursive predicates, recursive contracts
  - enter loop:
    - unfold predicates one step
    - compute DNF
    - simplify DNF
    - extract test-cases

    until we are satisfied, i.e. have „enough" test cases ...
  - Select test-data: constraint resolution of test cases.

Test-Data Generation

- Observation: "all other cases" ...
  were represented by the clauses still
  containing recursive predicates.
- Logically: we used a regularity hypothesis, i.e ...

$$(\forall X.\ |X|<k \Rightarrow X.sum() \equiv Y) \Rightarrow (\forall X.\ X.sum() \equiv Y)$$

where we choose as "complexity mesure" |X|
just X.lgth and k (the number of unfoldings)
was 2 ...

Test-Data Generation

- Coverage Criterion for recursive specification:

DNF_k

For all data up to complexity k, we constructed abstract test-cases and generated a test.

In our example, the "complexity measure" is just the length of the LLists.

Test-Data Generation

- What are the alternatives to symbolic test-case generation ?

  Must this really be so complicated ???

  Well, think about the probability to "guess" input with a complex invariant or precondition, if you use "blind" random-generation of input...

Test-Data Generation

- Summary
  - We have (sketched) a symbolic Test-Case Generation Procedure for UML/MOAL Specifications
  - It takes into account:
    - object orientation
    - data invariants (recursive predicates)
    - recursive functions (via unfolding)
  - The process can be tool-supported (HOL-TestGen)
  - The process is intended for automation.

Test-Data Generation

- Summary

Key-Ingredients are:

- Unfolding predicates up to a given depth k
- computing the Disjunctive Normal Form (DNF_k)
- Adequacy:
  Pick for each test-case (a conjoint in the DNF_k)
  one test, i.e. one substitution for the free
  variables satisfying the test-case !