# Interactive Theorem Proving and Applications

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`https://www.lri.fr/~wolff/teach-material/2023-2024/M2-CSMR/index.html`

## TP 1 - Introduction to Isabelle/HOL

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### Exercice 1 (Installation)

Install Isabelle(version 2023) from the Isabelle website `https://isabelle.in.tum.de`. Start Isabelle and make yourself familiar with the documentation, in particular the overview "What's in Main" where "Main" is the standard HOL library we base our exercises on.

### Exercice 2 (Editing, Type-Checking, Searching)

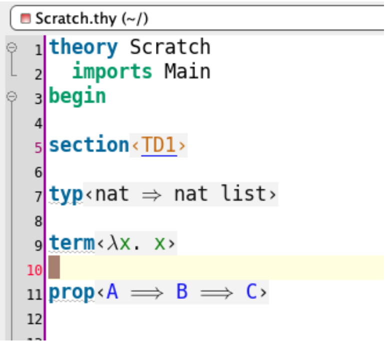
Use the Isabelle commands , `typ`, `term` and `prop` to enter types, terms and propositions into the system, thus using the parser and type-checker of Isabelle.

Start an anonymous session with `isabelle jedit` and create an anonymous session with

```
theory Scratch
  imports Main
begin
```

### Questions

1. Enter, parse and type-check (if possible) some types, terms, and propositions. The result should look similar to this : Change types, terms and props at your guise though.



```
theory Scratch
  imports Main
begin

section‹TD1›

typ‹nat ⇒ nat list›

term‹λx. x›

prop‹A ⟹ B ⟹ C›

```

FIGURE 1 – example caption

2. Enter, parse and type-check (if possible) the term

$$(\lambda x.\lambda y.(\lambda z.(\lambda x.z\ x)(\lambda y.z\ y))(x\ y))$$

(It might be helpful to add spaces ...) Note how the system represents bound and free variables.

3. Define via a number of `definition`s the Church Numerals of the slides of class 1. The syntax is :

$$\text{definition} \quad \mathit{const\_name} \quad :: \quad \mathit{typ} \quad \text{where} \quad {''}\mathit{eqn}{''}$$

   Which type do Church-Numerals have in the typed $\lambda$-calculus ?

4. axiomatize the Y-combinator, i.e. enter "Y f = f(Y f)" as axiom into the system. The syntax is :

$$\text{axiomatization} \quad \mathit{const\_name} \quad :: \quad \mathit{typ} \quad \text{where} \quad \mathit{ax\_name} : \quad {''}\mathit{eqn}{''}$$

   Which (external) type has to be given to the Y-combinator for this axiomatization ?

5. use `find_theorems` to browse your theory so far ! You will need this possibility later on !

6. Prove that, according to your definitions, $PLUS\ TWO\ THREE$ is indeed $FIVE$.
   Hint : state a `lemma` for this equation, unfold the definitions, and apply the simplification method `by simp`

**Exercice 3 (OPTIONAL : Report (IN CASE THAT YOU WANT TO HAVE IT GRADED :))**

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 3 pages max.