Interactive Theorem Proving and Applications

Année 2023-24

Prof. Burkhart Wolff wolff@lri.fr
`https://www.lri.fr/~wolff/teach-material/2023-2024/M2-CSMR/index.html`

TP 3 - Specification Constructs in Isabelle

Semaine du 22 janvier 2021

Exercice 1 (Datatypes and Simple Inductive Proofs)

1. Define your own version of the polymorphic "list"-data-type with the constructors `nil` and `snoc:: 'a list => 'a => 'a list` (the reverse cons).
2. Define the usual operations `filter`, `map` and `concat` on lists as recursive functions (Hint : use recursive `fun` definitions).
3. Prove : $filter\ p(filter\ q\ S) = filter(\lambda\ x.\ p\ x \wedge q\ x)S$
4. Prove : $map\ f\ (concat\ R\ S) = concat(map\ f\ R)(map\ f\ S)$
5. Prove : $map\ f(concat\ R\ S) = concat(map\ f\ R)(map\ f\ S)$

Exercice 2 (Inductive sets - Inductive Proofs)

Define the set of `Even` Integers (using the `Int` theory from the Main) inductively.

1. Either by the *specification construct* `inductive_set` or by `inductive` (predicate)
2. Prove : $4 \in Even$
3. Prove que $3 \notin Even$

Objective : try first elementary Isabelle proof methods, so i.e. `subst`, `rule`, `rule_tac`, `erule`, `erule_tac` before applying more advanced methods like `simp` and `auto`. Experiment with methods like `induct` and `cases` (See RefMan). At the end, try to find the most compact version possible.

Remark : A good balance between compactness and readability improves portability of your proof documents.

Exercice 3 (Modeling Exercise)

Define the λ-calcul type as a theory in HOL.

1. Define the “terms” (abstract syntax tree) of the untyped λ-calcul as “data type”
2. Define the “types” (abstract syntax tree) du λ-calcul as “data type”
3. Define a function `instantiate` for that substitutes type-variables against types.
4. The environments Σ et Γ by using the partial functions defined in the `Map.thy`-library providing the $'a \rightharpoonup\ 'b$ type.
5. Define inductively the well-typedness quartuple : a term t is well-typed with type τ in the environnements Σ et Γ.

Hints : Revise the slides of the cours *lambda calculus*,.

Exercice 4 (OPTIONAL : Report)
(IN CASE THAT YOU WANT TO HAVE IT GRADED. RECALL THAT 2 OUT OF 6 TP's SHOULD BE SUBMITTED.)

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 3 pages max (except screenshots and other figures).